sp(4,R)-systematics of atomic nuclei. F-multiplets and nuclear structure

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Abstract

A systematics of the atomic nuclei in the frame of the nucleon number $A = Z + N$ and the proton-neutron difference $F = Z - N$ is considered. The classification scheme is provided by means of the non-compact algebra $sp(4, R)$. In this scheme the nuclei are ordered into isobaric multiplets, for which $A =$ fix, as well as in F-multiplets, for which $F =$ fix. The dependence of the mass excess Δ, the first exited states E_{2^+} and the ratio $R_2 = E_{4^+}/E_{2^+}$ on the nucleon number A is empirically investigated within the F-multiplets. Appropriate filters are used to study the properties of the mass excess. Many structural effects are observed. The mirror symmetry is clearly indicated for the energy levels of the nuclei with the same value of A and opposite F-values.

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1 Introduction

Usually the chart of the atomic nuclei is imaged in the plane of the proton number Z and the neutron number N. It is naturally, because the protons and neutrons are the particles from which the nucleus is composed and the levels of the shell structure are given by their number. On the other hand there are important cases when it is suitable to use the nucleon number $A = Z + N$ and $F = Z - N$. For example: the Weizsäcker-Bethe mass formula [1]; the description of the stability line [2], [3], [4] the isospin symmetry, etc. Here, it is proposed to consider a systematics of the atomic nuclei in the framework of the atomic number $A = Z + N$ and the proton-neutron difference $F = Z - N$, together with (and not instead of) the systematics, based on Z and N. This idea is long known. First Ivan Selinov[5] made in 1948 a table of nuclei by using the coordinates A and $\frac{1}{2}(Z - N)$. This approach allows one to map the nuclei into the spaces of the two irreducible infinite oscillator representations of the non-compact algebra $sp(4, R)$. One can systematize the even-even and odd-odd nuclei (A - even) along the first one and the even-odd and odd-even nuclei (A - odd) along the order. The proposed systematics is suitable for study of the nuclear mass excess Δ and the half-life $T_{1/2}$. In particular, the behaviour of Δ as a function of F at $A =$ fix has the known parabolic form in a very wide interval (up to $A = 260$). In the case of isobaric multiplets with even A, the mass excess Δ, considered as a function of F, exhibits a staggering behaviour, corresponding to the alternation of the even-even and odd-odd nuclei. For even A isobaric multiplets with $A \leq 208$ and for odd A isobaric multiplets with $A \leq 209$ and $229 \leq A \leq 253$ both, the minimum of the mass excess Δ and the maximum of the half-life $T_{1/2}$, are at the same value of F. For the odd $211 \leq A \leq 227$ this rule is not fulfilled, while for the even $A \geq 210$ and the odd $A \geq 255$ the situation is ambiguous. The behaviour of Δ as a function on A for a given F-multiplet ($F =$ fix) is of a special interest. The corresponding curves $\Delta = \Delta(A)|_{F=\text{fix}}$ are examined directly, as well as with the help of their first and second discrete derivatives and also through a specially constructed discrete function. All considered curves show periodically repeating properties. All Z and N magic numbers giving the major shells are displayed by distinct changes in the behaviour of the analyzed curves. Also, a set of sub-magic numbers (giving sub-shells) as 6, 40, 64, etc is well seen. Noticeable changes in the behaviour of the curves are observed at other values of Z and N, such as 18, 60, 56, etc. They can be interpreted as signs of possible substructures. The common impression is that the curves $\Delta = \Delta(A)_{F=\text{fix}}$ together with the corresponding filters contain a lot of information which needs to be decoded and explained.

The dependence of the first exited states E_{2^+} and the ratio $R_2 = E_{4^+}/E_{2^+}$ of even-even nuclei on the atomic number A at $F =$ fix are empirically investigated. All major magic and doubly magic numbers are clearly displayed in the F-curves. Also a set of candidates for sub-magic numbers (giving sub-shells), especially $Z = 14, 16, 40$; $N = 14, 16, 38, 40$ is well seen. An interesting behaviour for the nuclei with $N = 88$: ${}^{154}_{66}Dy_{88}$, ${}^{152}_{64}Gd_{88}$, ${}^{150}_{62}Sm_{88}$, ${}^{148}_{60}Nd_{88}$, ${}^{146}_{58}Ce_{88}$, is observed.

The first four nuclei of this series are "left" neighbours in the corresponding F-curves of the nuclei ${}^{156}_{66}Dy_{90}$, ${}^{154}_{64}Gd_{90}$ ${}^{152}_{62}Sm_{90}$, ${}^{150}_{60}Nd_{90}$, which are considered as candidates for $X(5)$-nuclei (see [6] and [7]).

Table 1: **Shematic structure of the space H_+**

A\F	···	10	8	6	4	2	0	-2	-4	-6	-8	-10	···
0							□						
2						□	□	□					
4					□	□	□	□	□				
6				□	□	□	□	□	□	□			
8			□	□	□	□	□	□	□	□	□		
⋮	···	···	···	···	···	···	···	···	···	···	···	···	···

A symmetry of the excitation levels of mirror nuclei with respect to the inversion of the proton-neutron difference F is clearly observed: $E_{J^\pi}(A, F) = E_{J^\pi}(A, -F)$, where J and π are the angular momentum and the parity, respectively, of the states belonging to the same band [8].

2 $sp(4, R)$ - representations

In the space of the nucleon number $A = Z + N$ and the proton-neutron difference $F = Z - N \equiv 2T_0$ (T_0 is the third projection of the isotopic spin) the nuclear chart splits into two parts: 1) A and F are even; 2) A and F are odd. This splitting makes it possible to map the nuclei into the spaces H_+ and H_- of the two irreducible infinite oscillator representations of the non-compact algebra $sp(4, R)$ [9]. The nuclei with even A are mapped in H_+, while those with odd A are situated in H_-. This is illustrated schematically for the case of H_+ in Table 1.

In this scheme the nucleon number A is the first order Casimir operator of the "isobaric" compact subgroup $u(2) \subset sp(4, R)$. The nuclei are ordered in isobaric multiplets (isobars), corresponding to the irreducible representations of $u(2)$ given by the values of A (the rows in Table 1). Till now there are evidences for the existence of 294 isobars ($A = 1, ..., 294$). On the other hand, F is interpreted as the first order Casimir operator of the noncompact subgroup $u(1, 1) \subset sp(4, R)$. The values of F give the oscillator representations of $u(1, 1)$, according to which the nuclei are ordered in F-multiplets (the columns in Table 1). Till now there are evidences for the existence of 70 F-multiplets ($F = 8, ..., 1, 0, -1, ..., -61$).

3 F-multiplets

Let us consider the curves, giving the behaviour of the mass excess Δ as a function on A at $F =$ fix or in other words the behaviour of Δ within the F-multiplets. We shall refer to the corresponding curves as F-Δ-curves. In Fig. 1 and Fig. 2 examples of F-Δ-curves at $F = 0, -4, -23, -44$ are given. (All experimental data on the mass excess Δ, in MeV, are taken from [10].) These examples illustrate the main characteristics of the F-Δ -curves:

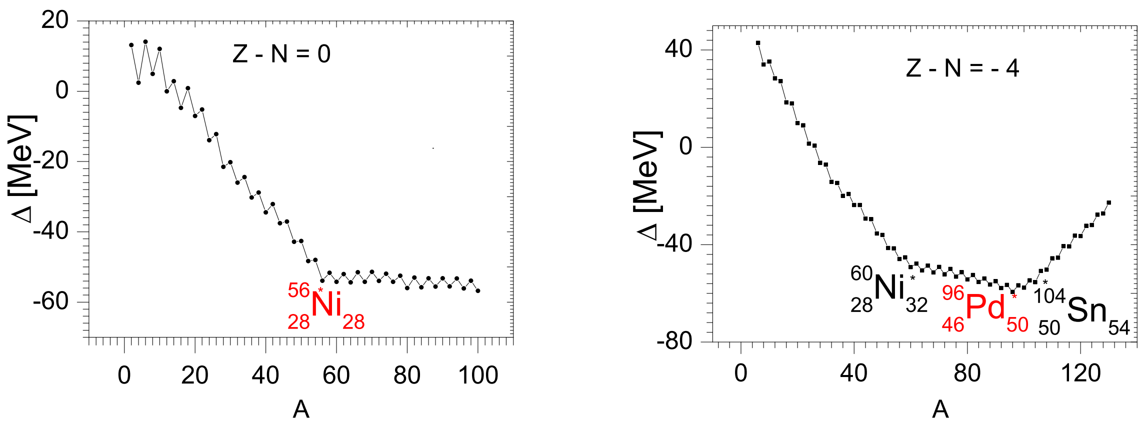

Figure 1: The mass excess Δ as a function of A for $F = 0$ (left) and $F = -4$ (right).

- The values of A for which the behaviour of a given F-Δ- curve changes considerably correspond to major magic Z-numbers or/and to major magic N-numbers. These are: ${}^{56}_{28}Ni_{28}$ at $F = 0$; ${}^{60}_{28}Ni_{32}$ and ${}^{96}_{46}Pd_{50}$ at $F = -4$; ${}^{123}_{50}Sn_{73}$ and ${}^{141}_{59}Pr_{82}$ at $F = -23$; ${}^{208}_{82}Pb_{126}$ at $F = -44$.

- The "odd" curves (F is odd) have relatively smoother behaviour while the "even" curves (F is even) have well seen sectors of a "staggering" and a "coupling" behaviour. It is known that analogical observation takes place in the case of isobaric multiplets. Note that this difference corresponds to the splitting of the nuclear chart into two subspaces, H_+ and H_-.

The staggering behaviour corresponds to alternating change up/down in the discrete-function value with the changing discrete values of the argument. In the case of even F the staggering corresponds to a splitting of the curve into two smoother curves, "even-even" and "odd-odd" ones. The coupling behaviour corresponds to alternating change short/long of the distance between two neighbouring points of a given sector of the discrete curve. In the case of even F the set of nuclei corresponding to this sector splits into couples each of them containing one even-even and one odd-odd nuclei.

- As a rule, when the curve has a clear minimum, this minimum corresponds to a nucleus with major magic Z or/and magic N (${}^{96}_{46}Pd_{50}$ at $F = -4$). From the other hand when F is even the minimum plays a role of a **reversal point** in which the order of the nuclei in the couple changes from (even-even,odd-odd) to (odd-odd,even-even) (There are exceptions, especially when the minimum is not distinctive.)

- In the case of the "even" F-Δ-curves the direction of the segments of the curve between the "coupled points" clearly changes at the values of A corresponding to nuclei with major magic Z or/and magic N numbers (e.g. ${}^{104}_{50}Sn_{54}$ at $F = -4$).

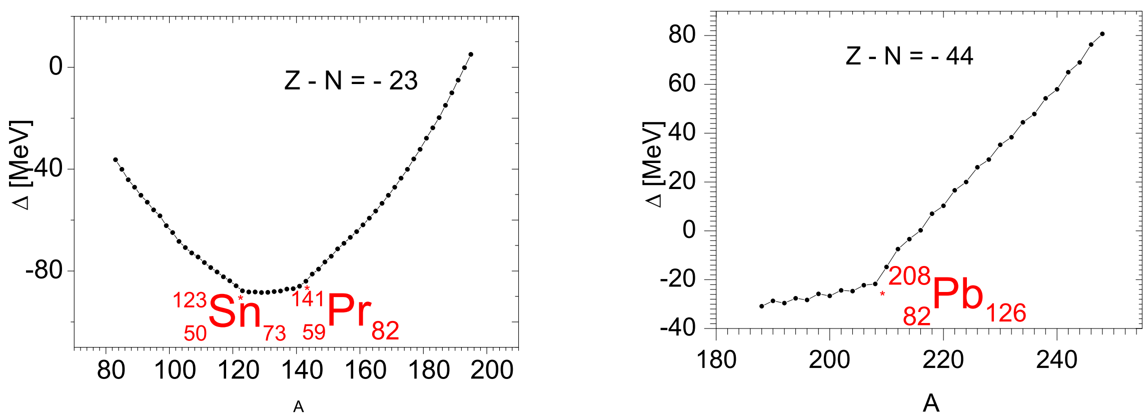

Figure 2: $\Delta(A)$ for $F = -23$ (left) and $F = -44$ (right).

However, such changes take place also at other values of A, pointing out the need of further study of the F-Δ-curves.

For more detailed investigations of the F-Δ-curves we introduce the following filters:

1. The first discrete derivative:

$$D_1[\Delta(A)] = 1/2[\Delta(A) - \Delta(A+2)]. \tag{1}$$

The staggering of the first discrete derivative corresponds to coupling and/or staggering behaviour of a given F-Δ-curve. When the staggering curve $D_1[\Delta(A)]$ does not cross the axis $\Delta = 0$, the curve has only a coupling behaviour. When the staggering curve $D_1[\Delta(A)]$ crosses the axis, the F-Δ-curve has certainly a staggering, which may be combined or not combined with the coupling behaviour. Compare Fig. 3, left with Fig. 3, right.

2. The modulus $|D_1[\Delta(A)]|$ of the first discrete derivative:

It clearly indicates the sector, where the staggering behaviour dominates ($A = 88, ..., 112$ at $F = -12$) and sectors with only coupling behaviour given by $88 \leq A$ and $A \geq 112$ at $F = -12$ (compare Fig. 3 (left) with Fig. 4 (left)). The function $|D_1[\Delta(A)]|$ indicates also **reversal points**, where the coupling is changed from "left to right" to "right to left" (at the reversal point the phase of the staggering changes).

3. The second discrete derivative:

$$D_2[\Delta(A)] = 1/4[\Delta(A+2) - 2\Delta(A) + \Delta(A-2)]. \tag{2}$$

It indicates both the coupling and the staggering effects, but can not distinguish them, when they take place simultaneously. From the other hand this filter is strongly sensitive to the deviations. As a rule these deviations are displayed at the values of A, which correspond to the magic or sub-magic numbers (see Fig. 4, right).

4. D-function:

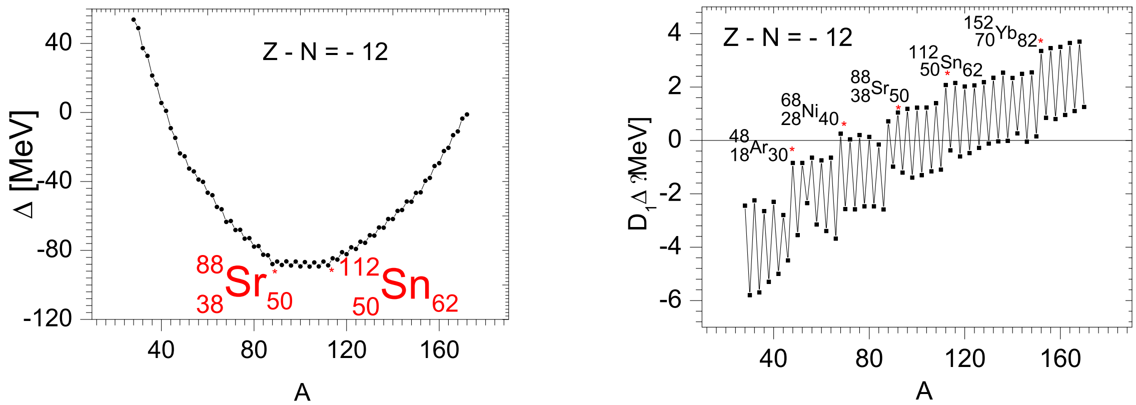

Figure 3: $\Delta(A)$ for $F = -12$ (left); $D_1\Delta(A)$ for $F = -12$ (right).

Let us introduce the function:

$$D[\Delta(A)] = |1/2([\Delta(A) - \Delta(A+2)] - [\Delta(A-4) - \Delta(A-2)])|, \tag{3}$$

for $A = A_0 + 4,\ A_0 + 8,\ \ldots\ , A_{min} - 4,$

$$D[\Delta(A)] = |1/2([\Delta(A) - \Delta(A-2)] - [\Delta(A+4) - \Delta(A+2)])|, \tag{4}$$

for $A = A_{min}, A_{min} + 4,\ A_{min} + 8,\ \ldots,\ A_f - 4,$
where A_0 and A_f are the values of A corresponding to the left point of the first couple of the F- Δ-curve and the right point of the last couple of the F-Δ - curve respectively; A_{min} is defined by $\Delta(A_{min}) = min\Delta(A)$.

The D-function indicates the changes of the direction of the segments between "coupled points" (see Fig. 5,left and right).

All filters are used here only for even multiplets.

The F-Δ- curves and the corresponding filters contain a lot of information:

- Clear indications for the existence of the shell structure of the nuclei are seen. All Z and N magic numbers giving the major shells are displayed by distinct changes in the behaviour of the F-Δ-curves and of the introduced filters

- A set of sub-magic numbers (giving sub-shells) as 40, 64, etc. is well seen.

- Noticeable changes in the behaviour of the filters under consideration are observed at other values of Z and N such as 18, 56, 60, etc. These effects need to be explained.

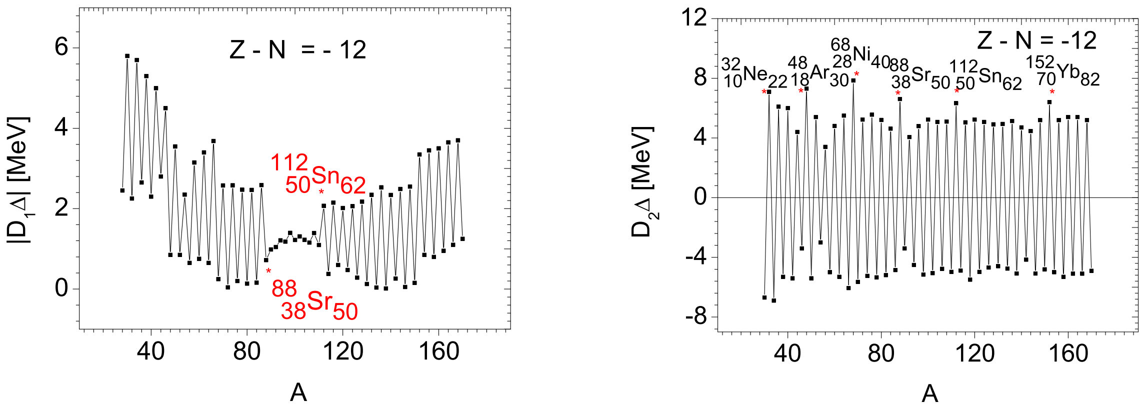

Figure 4: $|D_1[\Delta(A)]|$ for $F = -12$ (left); $D_2\Delta(A)$ for $F = -12$ (right).

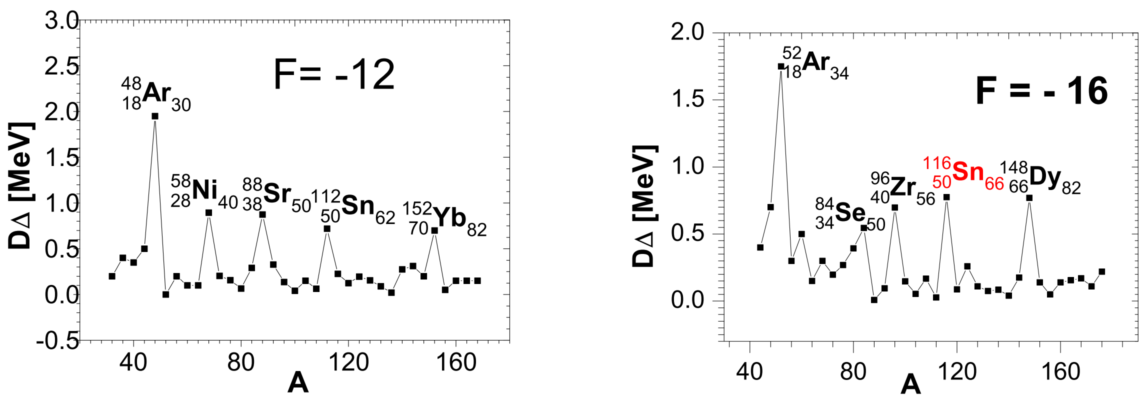

Figure 5: D-function for $F = -12$ (left); D-function for $F = -16$ (right).

4 F-E_{2_+}-curves and F-R_2-curves

In the case of the even-even nuclei a lot of information is contained also in the curves which give the dependence of E_{2_+} and $R_2 = E_{4_+}/E_{2_+}$ on A at F = fix. (E_{4_+} and E_{2_+} are from the ground band states.) We shall call these curves F-E_{2_+} - curves and F-R_2-curves respectively. The bundle of all known F-E_{2_+} - curves are displayed in Fig. 6. This picture is an other example, which shows the advantage of the systematics of the nuclei in F-multiplets:

- The major magic and doubly magic numbers are well seen as strong peaks of the F-E_{2_+}-curves and deep minimums of the F-R_2-curves. Some sub-magic nuclei as ${}^{96}_{40}Zr_{56}$ are also clearly displayed (see Fig. 6).

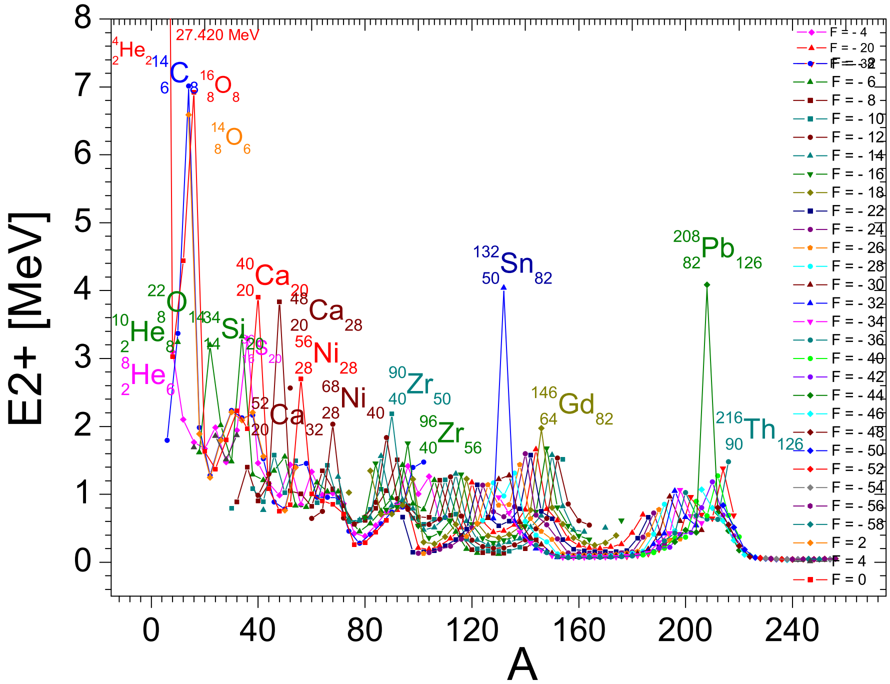

Figure 6: The behaviour of all F-E_{2_+}-curves. The experimental data are from [10].

- The even-even nuclei are grouped in shell multiplets. It is especially well seen for the major shells: $(28, 28|50, 50)$, $(28, 50|50, 82)$, $(50, 50|82, 82)$,; $(50, 82|82, 126)$, $(82, 126|126, ?)$. (We denote the given major shell with $(MZ_i, MN_j|MZ_k, MN_l)$, where MZ_i, MN_j and MZ_k, MN_l are two "neighbour" double magic numbers defining uniquely the shell ($MZ_i < MZ_k$, $MN_j < MN_l$).

An analogical analysis can be done for the bundle of all F-R_2-curves.

More detailed investigation can be provided if we consider the segments of the F-E_{2_+}-bundle and the F-R_2-bundle in a given major shell [11]. The picture of the segments of the F-E_{2_+}-bundle and the F-R_2-bundle in the framework of a given major shell presents a detached configuration. Usually, F-E_{2_+}-curves in a such segment decrease monotonously from the left side (inhabited by magic nuclei) to the bottom and after that go up monotonously to the right side (inhabited also by magic nuclei). In the case of heavy nuclei the "bottom" is flat in a very long interval. As to the F-R_2-curves, they increase monotonously from the left side (inhabited by magic nuclei) to the "roof" and after that go down monotonously to the right side (inhabited by magic nuclei also). In the case of heavy nuclei the "roof" is flat in a very long interval. In many large areas the curves do not intersect to each other, but the external curve embraces the internal one.

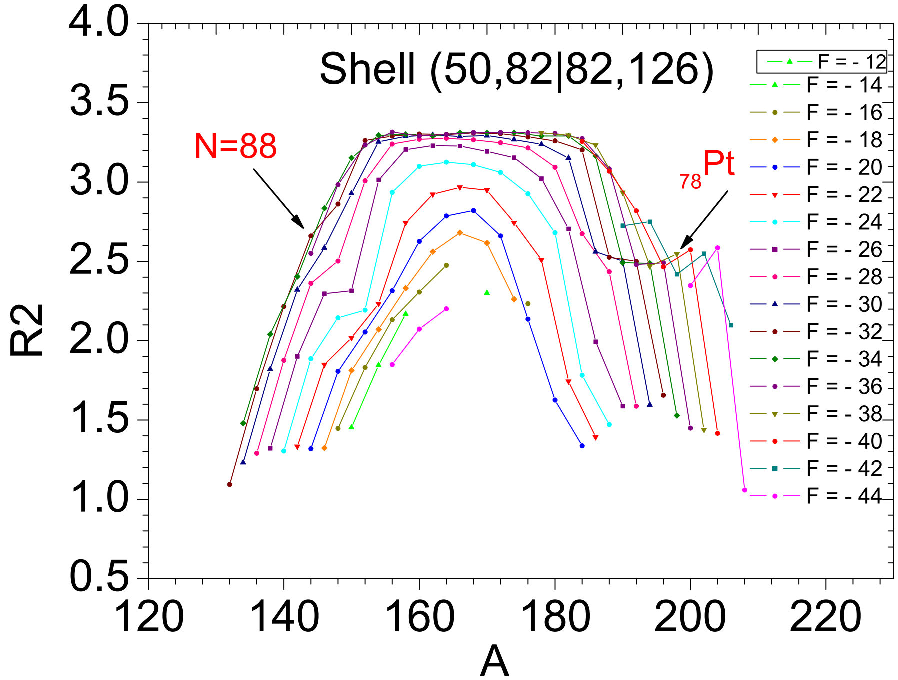

Figure 7: The behaviour in all F-multiplets passing through the shell $(50, 82|82, 126)$.

But, what we are looking for are the exceptions (the deviations) from this "right" behaviour. Let us point out several examples for deviations observed in different shells:

In the shell $(28, 28|50, 50)$ there are deviations at $F = 0, -2, -4, -6, -8$ and the nuclei:
${}^{70}_{30}Zn_{40}$. ${}^{64}_{32}Ge_{32}$, ${}^{66}_{32}Ge_{34}$, ${}^{68}_{32}Ge_{36}$,${}^{70}_{32}Ge_{38}$,${}^{72}_{32}Ge_{40}$
${}^{68}_{34}Se_{34}$,${}^{70}_{34}Se_{36}$, ${}^{72}_{34}Se_{38}$
${}^{72}_{36}Kr_{36}$,${}^{74}_{36}Kr_{38}$.

In the shell $(28, 50|50, 82)$ the strong deviations are observed at $F = -12, -14, -16, -18$ and the nuclei:
${}^{96}_{38}Sr_{58}$
${}^{92}_{40}Zr_{52}$, ${}^{94}_{40}Zr_{54}$, ${}^{96}_{40}Zr_{56}$, ${}^{98}_{40}Zr_{58}$,
${}^{96}_{42}Mo_{54}$, ${}^{98}_{42}Mo_{56}$.

A possible explanation of these deviations is an existence of a sub-shell. In the shell $(50, 50|82, 82)$ there are deviations at $F = -12, -14, -16, -18, -20$ and the nuclei:
${}^{144}_{66}Dy_{78}$, ${}^{142}_{64}Gd_{78}$, ${}^{140}_{62}Sm_{78}$, ${}^{138}_{60}Nd_{78}$, ${}^{136}_{58}Ce_{78}$ $N = 78$-series, (see Fig. 8, left and right).

In the shell $(50, 82|82, 126)$ there are deviations at
$F = -22, -24, -26, -28, -30$ and the nuclei: ${}^{154}_{66}Dy_{88}$, ${}^{152}_{64}Gd_{88}$, ${}^{150}_{62}Sm_{88}$, ${}^{148}_{60}Nd_{88}$, ${}^{146}_{58}Ce_{88}$ - $N = 88$-series (see Fig 7 and Fig 9, left).

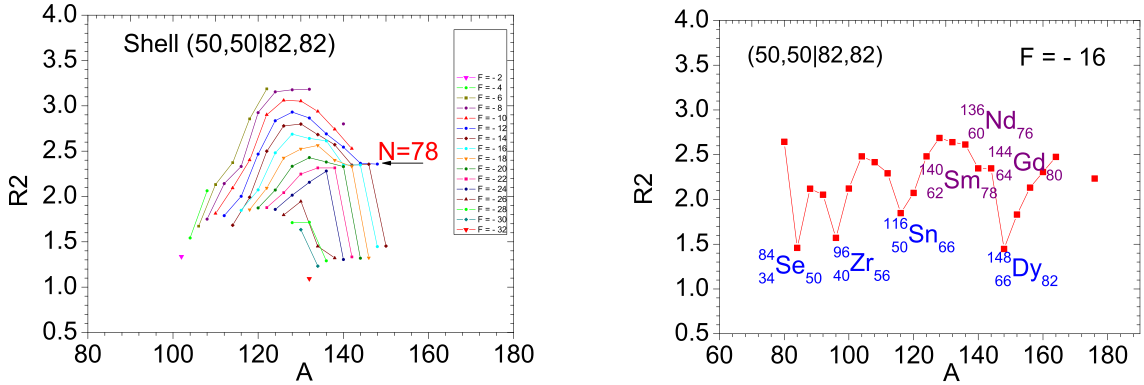

Figure 8: The bundle R_2-curves for all F-multiplets passing in the shell $(50, 50|82, 82)$ (left), the R_2-curve for $F = -16$ (right).

We remark that the first four nuclei of this series are "left" neighbours in the corresponding F-E_{2_+}-curves and F-R_2-curves of the nuclei ${}^{156}_{66}Dy_{90}$, ${}^{154}_{64}Gd_{90}$, ${}^{152}_{62}Sm_{90}$, ${}^{150}_{60}Nd_{90}$, which are considered as $X(5)$- nuclei [6] (see Fig. 9, left).

There are also well expressed deviations in this shell at $F = -28, -30, -32, -34, -36, -38, -40, -42$ and the nuclei: ${}^{184}_{78}Pt_{106}$, ${}^{186}_{78}Pt_{108}$, ${}^{188}_{78}Pt_{110}$, ${}^{190}_{78}Pt_{112}$, ${}^{192}_{78}Pt_{114}$, ${}^{194}_{78}Pt_{116}$, ${}^{196}_{78}Pt_{118}$, ${}^{198}_{78}Pt_{120}$ Pt-series (see Fig. 7 and Fig. 9, right).

Additional information can be extracted from F-E_{2_+}-curves and F-R_2-curves, each of them presented entirely (see, for example Fig. 8, right and Fig. 9, left and right). On these pictures all magic and doubly magic numbers are displayed clearly. Also, a set of candidates for sub-magic numbers (giving sub-shells) is well seen, especially $Z = 14, 16, 40$ and $N = 14, 16, 38, 40$.

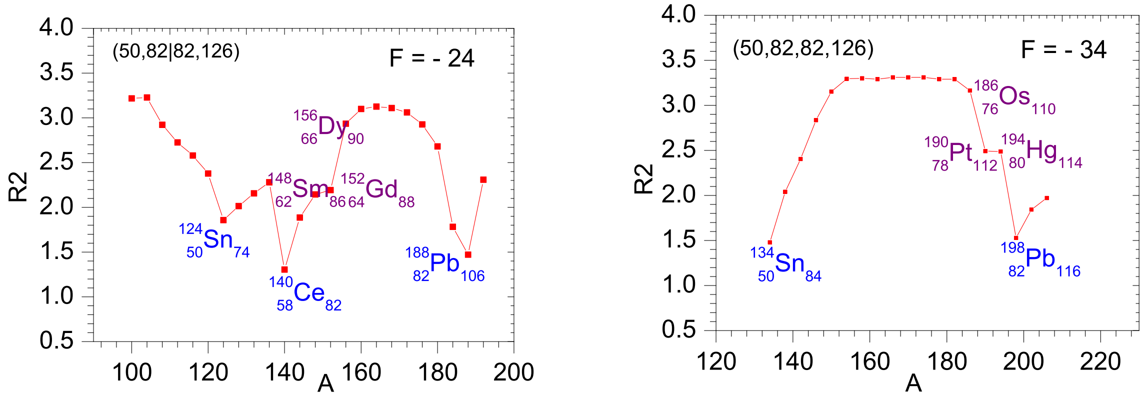

Figure 9: The behaviour of R_2-curves for the $F = -24$ (left) and for $F = -34$ (right).

5 F-multiplets and mirror nuclei

The F-multiplets are relevant also to study the properties of the mirror nuclei. Mirror nuclei are two nuclei with the same nucleon number A but interchanged proton number Z and neutron number N. They can be observed in the F-multiplets with $F = \pm 8, 7, 6, 5, 4, 3, 2, 1$. A symmetry is observed between the energy levels of the mirror nuclei: $E_{J^\pi}(A, F) = E_{J^\pi}(A, -F)$, where J is the angular momentum and π is the parity of the respective excited states of the same band. We examined this rule for all data in available which are at $F = \pm 1, ..., \pm 7$. The ground states of all mirror nuclei are at the same values of J^π. There are only three exceptions.These are the following ground states: ${}^{16}_{9}F_7(0^-)$ and ${}^{16}_{7}N_9(2^-)$ at $F = \pm 2$; ${}^{22}_{13}Al_9(3^+)$ and ${}^{22}_{9}F_{13}(4^+)$ at $F = \pm 4$; ${}^{25}_{15}P_{10}(1/2^+)$ and ${}^{25}_{10}Ne_{15}(3/2^+)$ at $F = \pm 5$.

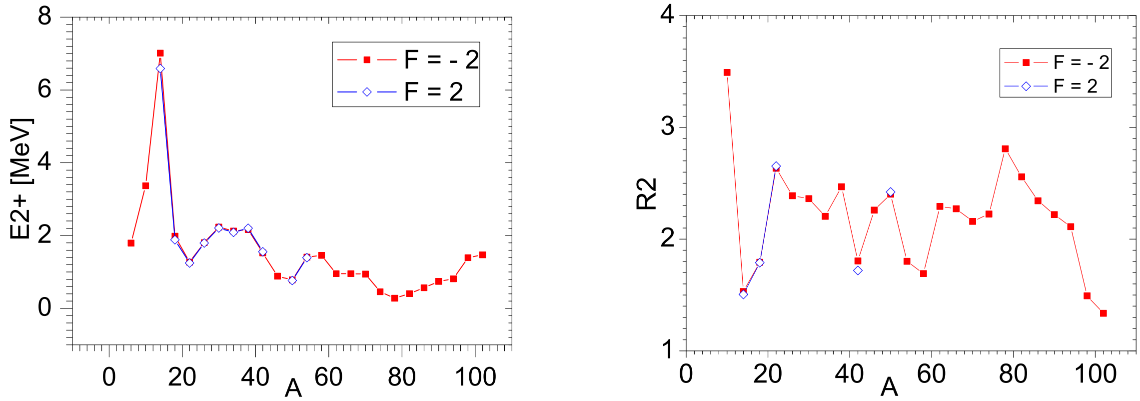

Figure 10: E_{2^+} energy states (left) and R_2-curves (right) for all mirror nuclei in $F = \pm 2$-multiplets.

The considered symmetry takes place for the data at $F = \pm 2, \pm 4$ and $J^\pi = +2^+, 4^+, ..., 10^+$ of the ground band. The F-E_{2^+} and F-R_2-curves at $F = \pm 2$ are given in Fig. 10. Another example is given in Fig. 11, left for $J^\pi = 1^+$ at $F = \pm 2$. The mirror energy symmetry is observed also for many states of the even-odd and odd-even nuclei belonging to the multiplets given by $F = \pm 1, \pm 3$. See for instance Fig. 11, right, for the case $J^\pi = 7/2^-$ at $F = \pm 1$. The mirror energy symmetry is suitable for the further investigation of the proton-rich nuclei.

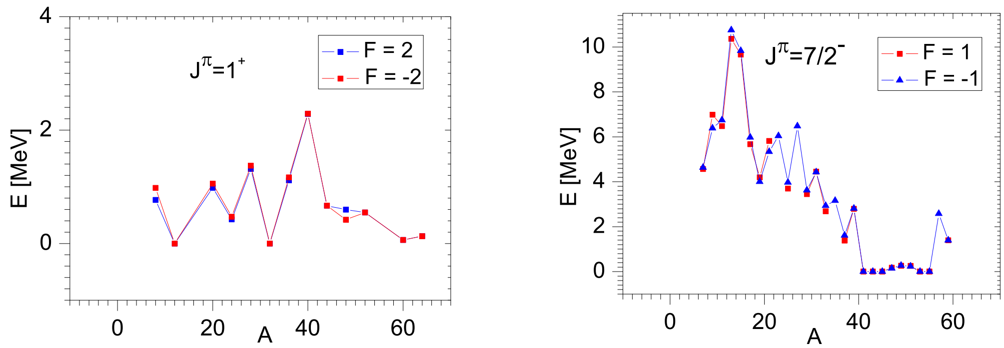

Figure 11: E_{1^+} energy states for all the mirror nuclei in $F = \pm 2$ -multiplets (left); $E_{7/2^-}$ energy states for all the mirror nuclei in $F = \pm 1$ (right).

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